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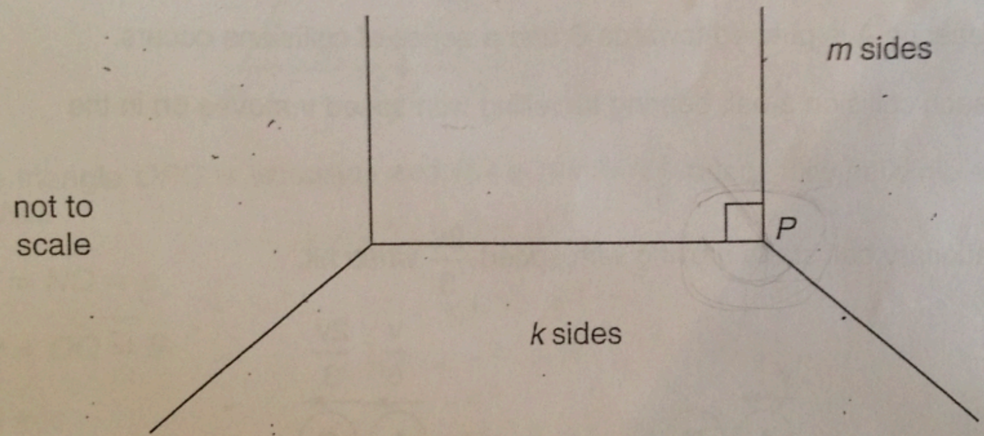
6: (i) Show that the size of the interior angle of a regular n-sided polygon may be written $\left(180 - \frac{360}{n}\right)^{\circ}$. (2)

Three regular polygons meet at point P. One polygon is a square and the others have k sides and m sides respectively.



(ii) Write down an equation involving k and m, using the fact that the angles at a point must add up to 360°. (2)

(iii) Show that the equation in part (ii) simplifies to $\frac{1}{4} = \frac{1}{k} + \frac{1}{m}$ (4)

(iv) Find all the possible pairs of values of k and m. (4)

i) The exterior angle of a regular polygon $= \frac{360}{\text{the number of sides}}$.
The interior angle and the exterior angle $= 180°$
Therefore $\left(180 - \frac{360}{n}\right)^{\circ}$ = the interior angle. ✓

ii) $\left(180 - \frac{360}{k}\right)^{\circ} + 90^{\circ} + \left(180 - \frac{360}{m}\right)^{\circ} = 360$ ✓

can you simplify this?

iii) $360 = \left(180 - \frac{360}{k}\right)^{\circ} + \left(180 - \frac{360}{m}\right)$

14. If a pair of letters is chosen from A, B and C, the possibilities are AB, AC and BC. (BA is considered to be the same as AB in this question.) Of these three possibilities only AC has non-adjacent letters of the alphabet.

(i) List the 3 pairs of non-adjacent letters which may be chosen from the first four letters of the alphabet A, B, C and D. (2)

(ii) List the 6 pairs of non-adjacent letters which may be chosen from the first five letters of the alphabet A, B, C, D and E. (2)

(iii) Calculate the number of pairs of non-adjacent letters which may be chosen from the first

(a) 6 letters of the alphabet (2)

(b) 7 letters of the alphabet. (2)

(iv) By finding a pattern to your answers, determine the number of pairs of non-adjacent letters which may be chosen when the entire alphabet of 26 letters is used. You may wish to use the fact that

$$1 + 2 + 3 + 4 \ldots\ldots + (n-1) + n = \tfrac{1}{2}n(n+1)$$ (3)

i) AC, BD, AD

ii) AC, BD, AD, BE, CE, AE

iii) 3 4 5 6 7 8 9 10 11

3 1 3 6 10 15 21 28 36

2 3 4 5 6 7 8

1 2 2

16. Barrel A contains wine and water in the ratio 3:2 and barrel B contains wine and water in the ratio 2:1

(i) 5 pints from barrel A and 6 pints from barrel B are mixed together.

Show that the ratio of wine to water in the mixture is 7:4 (2)

(ii) 10 pints from barrel A and 18 pints from barrel B are mixed together.

The ratio of wine to water in the mixture is x:5

Calculate the value of x. (2)

(iii) 20 pints from barrel A are mixed with y pints from barrel B, giving a mixture in which the ratio of wine to water is 5:3

Calculate the value of y. (5)

i) 5 Barrel A

5 pints = 3:2

3 Pints wine : 2 Pints water

Barrel B

6 = 2 : 1

iii) Barrel A

T.n.o.p = 5 3+2=5

1 part = 20/5 = 4

ratio = 12:8 ✓

Barrel B

T.n.o.p = ~~2y+y = 3y~~ 2+1=3

Calculate the value of x.

(iii) 20 pints from barrel A are mixed with y pints from barrel B, giving a mixture in which the ratio of wine to water is 5:3 (2)

Calculate the value of y. (5)

i) 5 Barrel A

5 pints = 3 : 2

3 Pints wine : 2 Pints water

Barrel B

6 = 2 : 1

T.n.o.p = 2+1 = 3

1 part = 6/3 = 2

4 Pints wine : 2 pints water

4 : 2

3 : 2

=

7 : 4 ✓

ii) 10 + 18 = 28

Barrel A

T.n.o.p = 3+2 = 5

1 part = 10/5 = 2

ratio = 6 : 4

Barrel B

T.n.o.p = 2 + 1 = 3

1 part = 18/3 = 6

ratio = ~~12 + 6~~ 12:6

Ratio = 12:6 + 6:4

= 18 : 10

÷2

9 : 5

x = 9 ✓

iii) barrel A

T.n.o.p = 5 3+2 = 5

1 part = 20/5 = 4

ratio = 12:8 ✓

Barrel B

T.n.o.p = ~~2y + y = 3y~~ 2+1 = 3

1 part = $y/3$

ratio = $\frac{2y}{3} : \frac{y}{3}$ ✓

12:8 $\frac{2y}{3} : y/3$

×3 ×3

36:24 2y : y

This must equal a multiple of 5 after 2y has been taken away from it.

together wine : water

$\left(12 + \frac{2y}{3}\right) : \left(8 + \frac{y}{3}\right)$

$\frac{36 + 2y}{\cancel{3}} : \frac{24 + y}{\cancel{3}}$

$36 + 2y : 24 + y$

to equal the ratio 5:3

35 : 21

40 : 24

45 : 27 2y = 11 & y = 3 ×

50 : 30 2y = 14 & y = 6 ×

55 : 33 2y = 19 & y = 9 ×

60 : 36 2y = 24 y = 12 ✓

18 In ancient Egypt the only fractions used were those with a numerator of 1, called unit fractions, for example $\frac{1}{3}, \frac{1}{15}, \frac{1}{401}$

They could therefore write $\frac{3}{5}$ as $\frac{1}{2}\ \frac{1}{10}$ since $\frac{3}{5} = \frac{1}{2} + \frac{1}{10}$

and $\frac{5}{11}$ as $\frac{1}{3}\ \frac{1}{11}\ \frac{1}{33}$ since $\frac{5}{11} = \frac{1}{3} + \frac{1}{11} + \frac{1}{33}$

When converting a fraction into Egyptian form, the unit fractions must not be repeated, eg you cannot write $\frac{3}{7} = \frac{1}{7}\ \frac{1}{7}\ \frac{1}{7}$

(a) Show that $\frac{11}{18} = \frac{1}{3}\ \frac{1}{6}\ \frac{1}{9}$ (2)

(b) Find the ordinary fraction equal to $\frac{1}{3}\ \frac{1}{4}\ \frac{1}{5}\ \frac{1}{6}$ (3)

(c) Put into Egyptian form the fraction $\frac{5}{9}$ (4)

(d) Find two ways of expressing $\frac{7}{13}$ in Egyptian form. (5)

a) $\frac{11}{18} = \frac{1}{3}\ \frac{1}{6}\ \frac{1}{9}$ d)

First we need make them all have a denominator of 18

$\frac{1}{3}\ \frac{1}{6}\ \frac{1}{9}$